

Chapter 11

Solution of Differential Equations with MATLAB

MATLAB has some powerful features for solving differential equations of all types. We will explore some of these features for the CCODE forms. The approach here will be that of the Symbolic Math Toolbox. The result will be the form of the function and it may be readily plotted with MATLAB.

Symbolic Differential Equation Terms

 y y $\frac{dy}{dt}$ Dy $\frac{d^2 y}{dt^2}$ D^2y $\frac{d^n y}{dt^n}$ $D^n y$

Representative CCLODE Form

$$b_2 \frac{d^2 y}{dt^2} + b_1 \frac{dy}{dt} + b_0 y = A \sin at$$

$$y(0) = C_1 \quad \text{and} \quad y'(0) = C_2$$

```
>> y = dsolve('b2*D2y+b1*D1y+b0*y=A*sin(a*t)',  
              'y(0)=C1', 'Dy(0)=C2')
```

```
>> ezplot(y, [t1 t2])
```

Example 11-1. Solve DE below with MATLAB.

$$\frac{dy}{dt} + 2y = 12 \quad y(0) = 10$$

```
>> y = dsolve('Dy + 2*y = 12', 'y(0)=10')
```

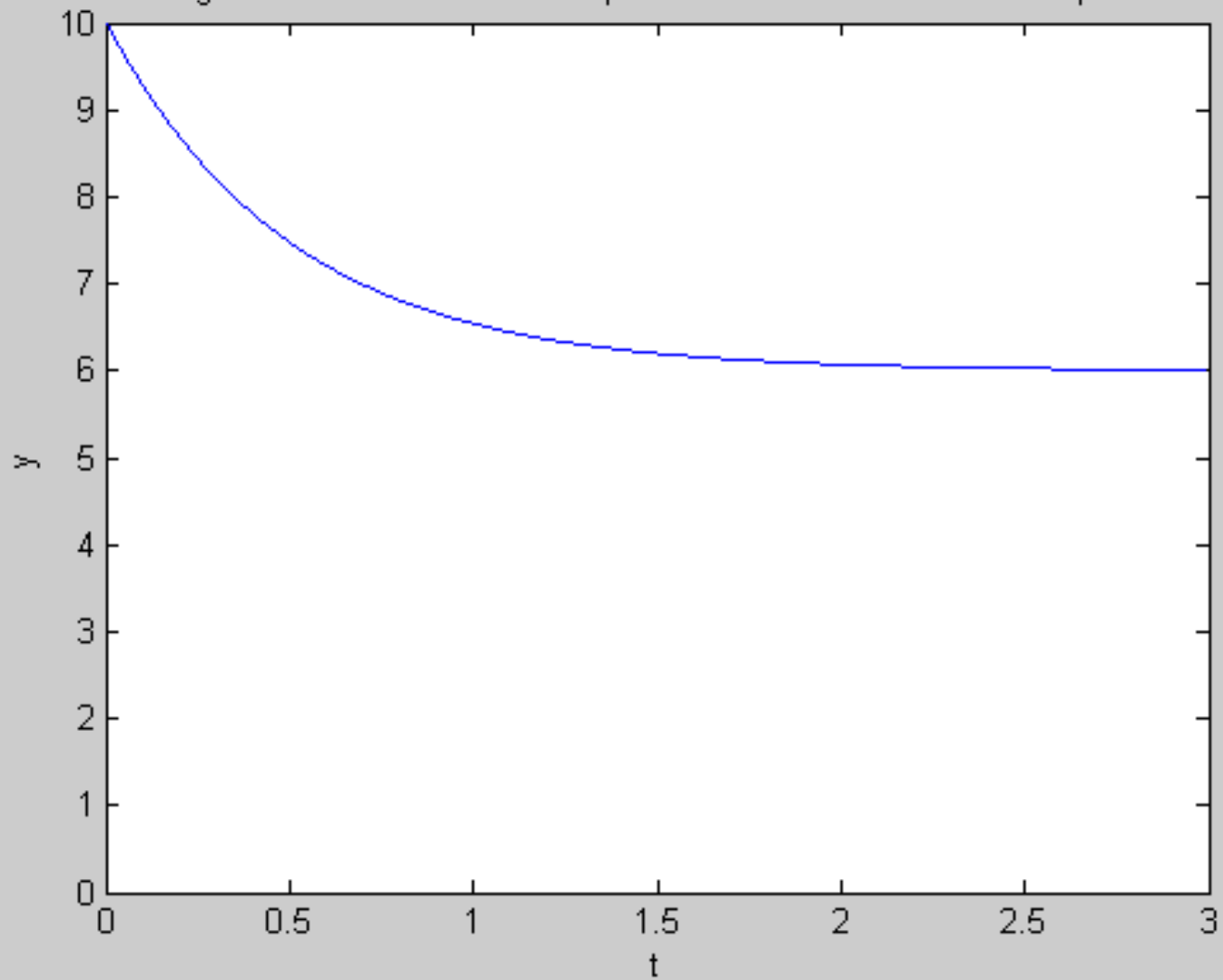
```
y =
```

```
6 + 4*exp(-2*t)
```

```
>> ezplot(y, [0 3])
```

```
>> axis([0 3 0 10])
```

Figure 11-1. Solution of Example 11-1 based on dsolve and ezplot.



Example 11-2. Solve DE below with MATLAB.

$$\frac{dy}{dt} + 2y = 12 \sin 4t \quad y(0) = 10$$

```
>> y = dsolve('Dy + 2*y = 12*sin(4*t)',  
             'y(0)=10')
```

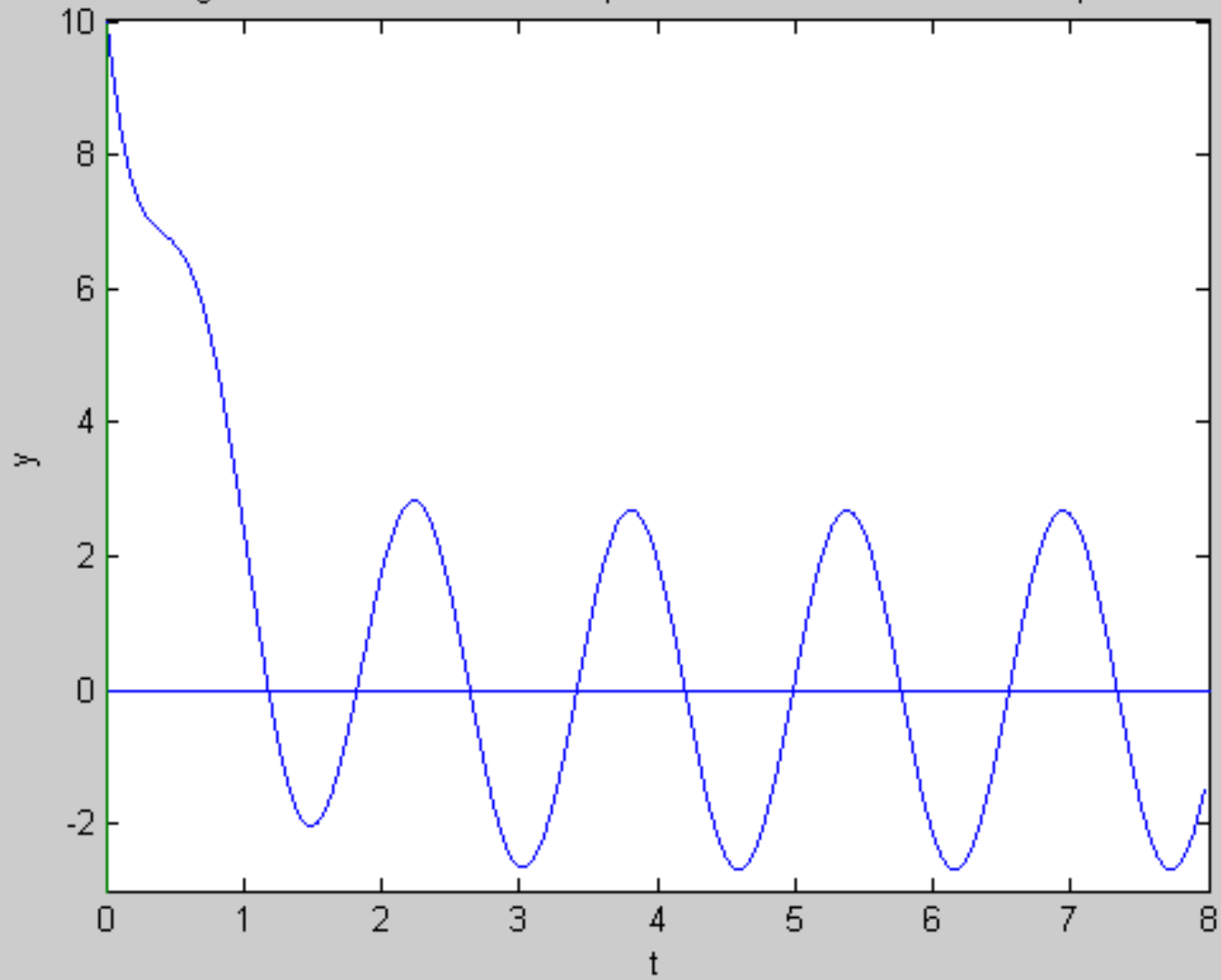
```
y =
```

```
-12/5*cos(4*t) + 6/5*sin(4*t) + 62/5*exp(-2*t)
```

```
>> ezplot(y, [0 8])
```

```
>> axis([0 8 -3 10])
```

Figure 11-2. Solution of Example 11-2 based on dsolve and ezplot.



Example 11-3. Solve DE below with MATLAB.

$$\frac{d^2 y}{dt^2} + 3\frac{dy}{dt} + 2y = 24$$

$$y(0) = 10 \quad y'(0) = 0$$

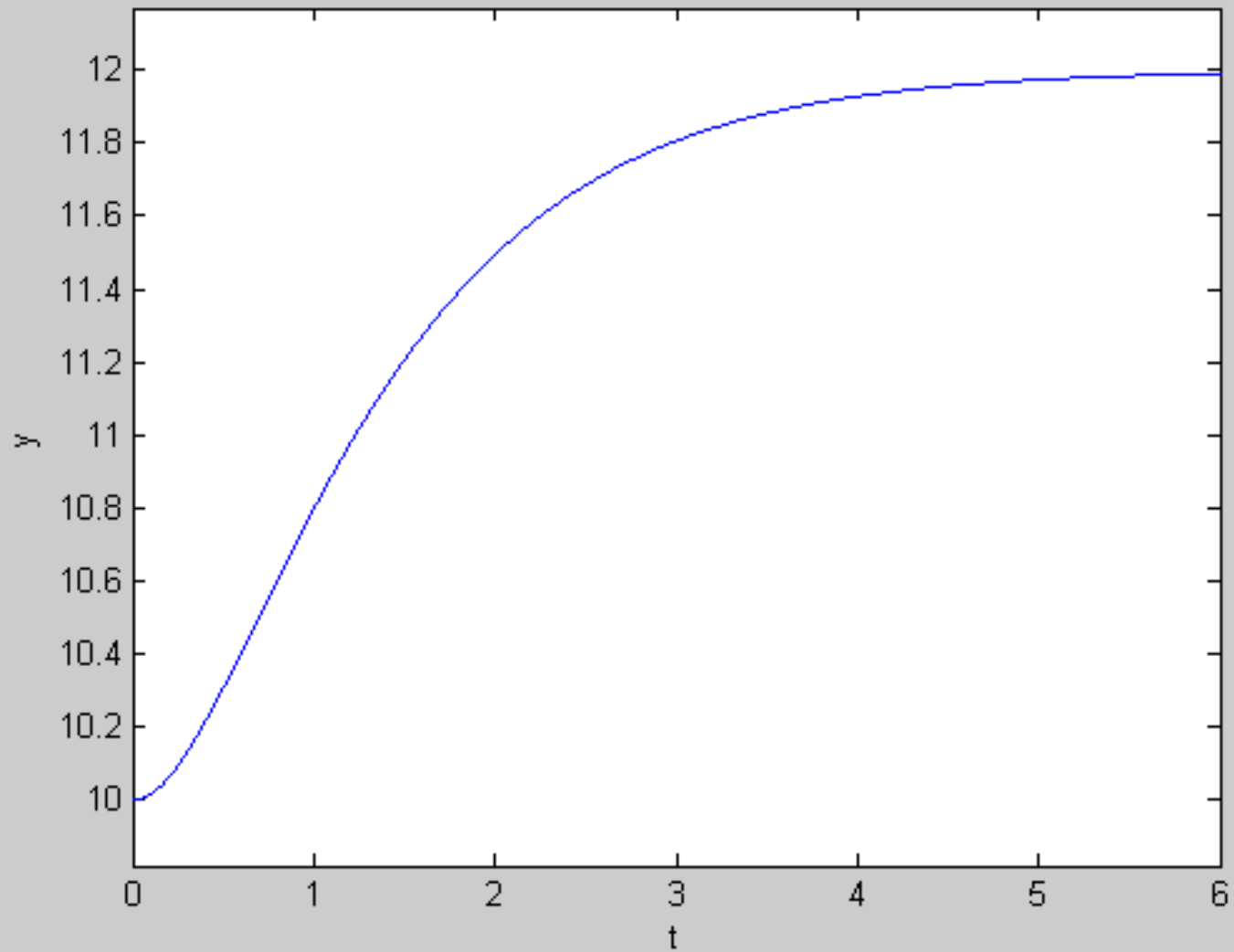
```
>> y = dsolve('D2y + 3*Dy + 2*y = 24',  
              'y(0)=10', 'Dy(0)=0')
```

```
y =
```

```
12 + 2*exp(-2*t) - 4*exp(-t)
```

```
>> ezplot(y, [0 6])
```


Figure 11-3. Solution of Example 11-3 based on dsolve and ezplot.



Example 11-4. Solve DE below with MATLAB.

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 5y = 20$$

$$y(0) = 0 \quad y'(0) = 10$$

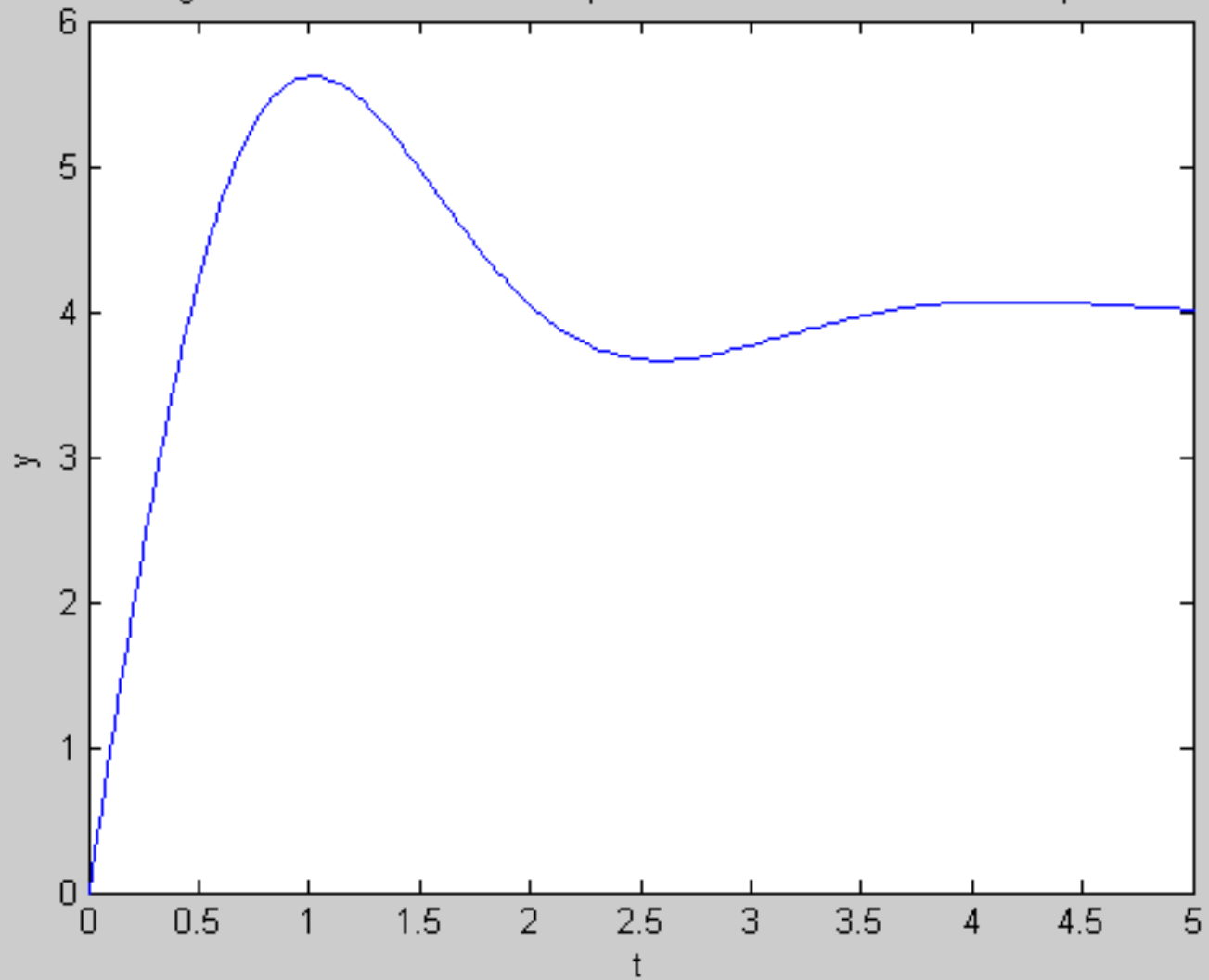
```
>> y = dsolve('D2y + 2*Dy + 5*y = 20',  
              'y(0) = 0', 'Dy(0) = 10')
```

```
y =
```

```
4 + 3*exp(-t)*sin(2*t) - 4*exp(-t)*cos(2*t)
```

```
>> ezplot(y, [0 5])
```

Figure 11-4. Solution of Example 11-4 based on dsolve and ezplot.



Symbolic Laplace Transform

Establish t and s as symbolic variables.

```
>> syms t s
```

The time function f is then formed and the Laplace transform command is

```
>> F = laplace(f)
```

Some useful simplifications are

```
>> pretty(F)
```

```
>> simplify(F)
```

Symbolic Inverse Laplace Transform

Establish t and s as symbolic variables.

```
>> syms t s
```

The Laplace function F is then formed and the inverse Laplace transform command is

```
>> f = ilaplace(F)
```

The simplification operations may also be useful for inverse transforms.

Example 11-5. Determine the Laplace transform of $f(t) = 5t$ with MATLAB.

```
>>syms t s
```

```
>> f = 5*t
```

```
f =
```

```
5*t
```

```
>> F = laplace(f)
```

```
F =
```

```
5/s^2
```

Example 11-6. Determine the Laplace transform of the function below using MATLAB.

$$v(t) = 3e^{-2t} \sin 5t + 4e^{-2t} \cos 5t$$

```
>> syms t s
```

```
>> v = 3*exp(-2*t)*sin(5*t)  
      + 4*exp(-2*t)*cos(5*t)
```

```
v =
```

```
3*exp(-2*t)*sin(5*t) + 4*exp(-2*t)*cos(5*t)
```

Example 11-6. Continuation.

```
>> V = laplace(v)
```

```
V =
```

$$15/((s+2)^2+25) + 4*(s+2)/((s+2)^2+25)$$

```
>> V=simplify(V)
```

```
V =
```

$$(23+4*s)/(s^2+4*s+29)$$

Example 11-7. Determine the inverse transform of the function below using MATLAB.

$$F(s) = \frac{100(s+3)}{(s+1)(s+2)(s^2+2s+5)}$$

```
>> syms t s
```

```
>> F=100*(s+3)/((s+1)*(s+2)*(s^2+2*s+5))
```

```
F =
```

```
(100*s+300)/(s+1)/(s+2)/(s^2+2*s+5)
```

Example 11-7. Continuation.

```
>> f = ilaplace(F)
```

```
f =
```

```
50*exp(-t)-20*exp(-2*t)-30*exp(-t)*cos(2*t)-  
10*exp(-t)*sin(2*t)
```

```
>> pretty(f)
```

```
50 exp(-t) - 20 exp(-2 t) - 30 exp(-t) cos(2 t)  
- 10 exp(-t) sin(2 t)
```

Example 11-8. Determine the inverse transform of the function below using MATLAB.

$$Y(s) = \frac{10}{s+2} + \frac{48}{(s+2)(s^2+16)}$$

```
>> syms t s
```

```
>> Y = 10/(s+2) + 48/((s+2)*(s^2+16))
```

```
Y =
```

```
10/(s+2) + 48/(s+2)/(s^2+16)
```

Example 11-8. Continuation.

```
>> y = ilaplace(Y)
```

```
y =
```

```
62/5*exp(-2*t)-
```

```
12/5*cos(16^(1/2)*t) + 3/10*16^(1/2)*sin(16^(1/2)*t)
```

```
>> y=simplify(y)
```

```
y =
```

```
62/5*exp(-2*t)-12/5*cos(4*t) + 6/5*sin(4*t)
```